the torus, and let the circle $H' f' L'$, and the rectangle H I M L be the analogous projections of the cylinder, which passes perpendicularly through it. If now we conceive, as before, a plane $a\ b$, Fig 13, to pass horizontally through both solids, it will obviously cut the cylinder in a circle which will be projected in the base $H' f' L'$ itself, and the ring in two other circles of which one only, part of which is represented by the arc $f'\ b^3\ b'$, will intersect the cylinder at the points f' and b^3, which being projected vertically to Fig 13, will give two points f and b^2 in the upper curve of penetration.

Another horizontal plane, taken at the same distance *below* the centre line A B, as that marked $a\ b$ is above it, will evidently cut the ring in circles coinciding with those already obtained; consequently the points f' and b^3 indicate points in the lower, as well as in the upper curves of penetration, and are projected vertically at d and e. Thus we find that by laying down two planes at equal distances on each side of A B, we obtain, by one operation, four points in the curves required.

In order to determine the vertices m and n, we must again have recourse to the method explained in the preceding problem; that is, to draw a plane $O'\ n$, passing through the axis of the cylinder and the centre of the ring, and to conceive this plane to be moved round the point O', as on a hinge, until it has assumed the position $O'\ B'$, parallel to the vertical plane; the point h', representing the extreme outline of the cylinder in plan, will now be at r', and being projected vertically, that outline will cut the ring in two points p and r, which would be the limits of the curves of penetration in the supposed relative position of the two solids; and by drawing the two horizontal lines $r\ n$ and $p\ m$, and projecting the point n' vertically, we obtain, by the intersections of these lines, the two points m and n, which are the vertices of the curves in the actual position of the penetrating bodies.

The points at which the curves are tangents to the outlines H I and L M of the cylinder, may readily be found by describing arcs of circles from the centre O' through the points H and L', which represent these lines in the plan, and then proceeding, as above, to project the points thus obtained upon the elevation. Lastly, to determine the points, as j, z, &c., where the curves are tangents to the horizontal outlines of the ring, draw a circle $P'\ s'\ j'$ with a radius equal to that of the centre line of the ring, namely P D; the points of intersection z' and j' are the horizontal projections of the points sought.

Required to represent the sections which would be made in the ring now before us, by two planes, one of which, $N'\ T'$, *is parallel to the vertical plane, while the other* $T'\ E'$ *is perpendicular to both planes of projection.*

The section made by the last-named plane must obviously have its vertical projection in the line C D, which indicates the position of the plane;* but the former will be represented in its actual form and dimensions in the elevation. To determine its outlines, let two horizontal planes $g\ q$ and $i\ k$, equidistant from the centre line A B, be supposed to cut the ring; their lines of intersection with it will have their horizontal projections in the two circles $g'\ o'$ and $h'\ q'$ which cut the given plane N T in o' and q'. These points being projected vertically to o, q, k, &c., give four points in the curve required. The line $N'\ T'$ cutting the circle $A'\ E'\ B'$ at N', the projection N of this point is the extreme limit of the curve.

The circle $P\ s'\ j'$, the centre line of the rim of the torus, is cut by the planes $N'\ T'$ at the point s', which being projected vertically upon the lines D P and C l, determines s and l, the points of contact of the curve with the horizontal outlines of the ring. Finally, the points, t and u are obtained by drawing from the centre O, a circle $T'\ v'$ tangent to the given plane, and projecting the point of intersection v' to the points v and x, which are then to be replaced upon C D by drawing the horizontals $v\ t$ and $x\ u$.

Penetrations of Cylinders, Prisms, Spheres, and Cones.—Plate VI.

Figs. 15 and 16.—*Required to delineate the lines of penetration of a sphere and a regular hexagonal prism whose axis passes through the centre of the sphere.*

The centres of the circles forming the two projections of the sphere are, according to the terms of the problem, upon the axis C C of the upright prism, which is projected horizontally in the regular hexagon $D'\ E'\ F'\ G'\ H'\ I'$. Hence it follows, that as all the lateral faces of the prism are equidistant from the centre of the sphere, their lines of intersection with it will necessarily be circles of equal diameters. Now, the perpendicular face represented by the line $E'\ F'$ in the plan, will meet the surface of the sphere in two circular arcs E F and L M, Fig. 15, described from the centre C, with a radius equal to $c'\ b'$ or $a'\ c'$. And the intersections of the two oblique faces $D'\ E'$ and $F'\ G'$ will obviously be each projected in two arcs of an ellipse whose major axis $d\ g$ is equal to the diameter of the circle $a\ c\ b$, and the minor axis is the vertical projection of that diameter, as represented at $e'\ f'$, Fig. 16. But as it is necessary to draw small portions only of these curves, the following method may be employed.

It is sufficiently evident that the horizontal line D G will pass through all the points where the edges of the visible faces of the prism intersect the surface of the sphere. Now, if we divide the portions E F and F G respectively into the same number of equal parts, and, drawing perpendiculars through the points of division, set off from F G the distances from the corresponding points in E F to the circular arc $E'\ c$ F, we shall have as many points in the elliptical arc required as we have taken divisions upon the chords. The remaining ellip-

* We may here observe, that in drawing the details of machinery, particularly if on a large scale or of the actual size, a mode of representation similar to that exemplified in Fig. 13, is frequently resorted to, and is attended with considerable economy of time and space. When the form of a piece is exactly symmetrical on either side of a centre line, it is sufficient for all practical purposes to represent on one side of the centre line, half the external elevation of the object, and on the other, half the section or plan, according as its nature or form may require.